ELSEVIER

Optical Materials 14 (2000) 217–221

OPTICAL Materials

www.elsevier.nl/locate/optmat

Analysis of characteristics of vertical-cavity surface-emitting lasers with a modified rate equation

Junfeng Song *, Yanping Fu, Yang Liu, Yuchun Chang, Bonan Kang, Jingzhi Yin, Xuemei Li, Guotong Du

Department of Electronic Engineering, State Key Laboratory on Integrated Optoelectronics, Jilin University, Jiefang Road 125, Changchun 130023, People's Republic of China

---

Abstract

By introducing a term of longitudinal coupling factor, we get a modified rate equation for vertical-cavity surface-emitting laser (VCSEL). Also by employing it, we give the expression for threshold current, and deduce from it that the threshold current decreases when the longitudinal coupling factor increases. For MQWs structure, the relation between cavity loss and the optimal quantum-well (QW) number is studied. We also analyze the small-signal modulation property of MQWs as well as periodic gain structure. Results show that, periodic gain structure gets advantages over general structures of VCSEL. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 78.20.Bh; 78.66.-w; 78.20.Ci

Keywords: VCSEL; Rate equation; Longitudinal coupling factor

---

1. Introduction

Vertical-cavity surface-emitting laser (VCSEL) has become one of the most active topics in present optoelectronics for its attractive properties, such as low threshold, small divergent angle, easiness to couple with fiber and the readily permission for large area and high density 2-D array [1,2]. Since VCSEL was first proposed in 1977, great progresses have been made on it. VCSEL devices of different characteristics have appeared, for example, devices of low threshold [3], room-temperature continuous-wave operation and wavelength of different domain from visible to infrared, etc. [4–6]. What is the same for VCSEL and conventional edge emitting laser is that both have a resonator, where interference generates crest and trough and optical field takes the form of a standing wave. Still they are quite different. For conventional edge emitting lasers, the propagation direction of optical field is parallel to the heterojunction plane, so there exist both crest and trough in the active region. When taken as a whole, as a result of average, the cavity gain is of the same value anywhere in the cavity, and has nothing to do with the position of the standing wave. While for VCSEL, the propagation of optical field is vertical to the heterojunction plane, so generally only a part of the standing wave is on the gain way for its thin active

---

* Corresponding author.

E-mail address: laserlab@mail.jlu.edu.cn (J. Song).

0925-3467/00/$ - see front matter
PII: S0925-3467(99)00140-8

region. Obviously only when the active region lies in the crest of the interference optical field, we can get the best coupling between carriers and optical field in the longitudinal direction. Also when the structure of active region and its position are changed, the longitudinal coupling may change dramatically, and in turn affect the performance and properties of VCSEL. This gets its full expression in periodic gain structure of VCSEL [7,8]. For VCSEL, we believe that it is necessary to introduce the longitudinal coupling into the rate equation instead of using the rate equation for conventional edge emitting lasers [9]. By employing the modified rate equations, we discuss the effect of interference on threshold current as well as the optimal number of quantum wells (QWs). In periodic gain structure, we also discuss the effects of the number of QWs on the property of small-signal modulation.

2. Foundation of the rate equations of travelling waves

As we know, the square of the intensity of the optical field is directly proportional to the number of photons, the interference of optical fields can be taken as that of the photons. In the active region, photons experienced interference interact with carriers and could be divided into left- and right-travelling waves. We assume that the optical field is allocated by the ratio between the photons of left- and right-travelling waves. Also the spontaneous emission distributes photons equally into the two opposite travelling waves. At the same time, we assume that the QW is narrow enough so that carrier in it is evenly distributed. We then get the following equations for travelling waves:

In the ith QW,

$$\begin{aligned}\frac{\partial p_i^+}{\partial t} + v_g\frac{\partial p_i^+}{\partial z} &= v_g(\Gamma_t g_i - \alpha_w)\left(1 + \frac{2\sqrt{p_i^+ p_i^-}}{p_i^+ + p_i^-}\cos(\theta(z))\right)p_i^+ \\ &\quad + \Gamma_t\beta\frac{n_i}{2\tau_n}, \qquad (1)\end{aligned}$$

$$\begin{aligned}\frac{\partial p_i^-}{\partial t} - v_g\frac{\partial p_i^-}{\partial z} &= v_g(\Gamma_t g_i - \alpha_w)\left(1 + \frac{2\sqrt{p_i^+ p_i^-}}{p_i^+ + p_i^-}\cos(\theta(z))\right)p_i^- \\ &\quad + \Gamma_t\beta\frac{n_i}{2\tau_n}, \qquad (2)\end{aligned}$$

$$\begin{aligned}\frac{\partial n_i}{\partial t} &= \frac{J}{e\sum_i d_i} - \frac{n_i}{\tau_n} - \frac{v_g g_i}{d_i}\int_{\mathrm{act}_i} p_i^+ + p_i^- \\ &\quad + 2\sqrt{p_i^+ p_i^-}\cos(\theta(z))\,\mathrm{d}z \qquad (3)\end{aligned}$$

and outside the QW,

$$\begin{aligned}\frac{\partial p^\pm}{\partial t} \pm v_g\frac{\partial p^\pm}{\partial z} &= v_g(-\alpha_b)\left(1 + \frac{2\sqrt{p^+ p^-}}{p^+ + p^-}\cos(\theta(z))\right)p^\pm, \qquad (4)\end{aligned}$$

where p^+ stands for the right-travelling wave, and p^- for the left-travelling wave. v_g represents the group velocity, equal to c/n_g, while c the velocity of light, n_g the group index. Γ_t is the transversal light confinement factor, which is determined by the active region. g is the gain, α_w the absorption coefficient in QW, α_b the outside QW, $\theta(z)$ the phase difference between the two opposite travelling waves, $\theta = 2k(z - z_0) + \theta_0$, when $z = z_0$, θ values at θ_0, where k is the wave vector in dielectric material. β is the spontaneous emission factor, n_i the density of the carriers in the ith QW. τ_n is the life time of carrier, and J is the surface density of the injection current, d_i the width of the ith QW, L the cavity length, n_N the transparent carrier density, and g_0 is the gain coefficient. In the foundation of these equations, we neglect the effect of the index difference between QW and the barrier on the reflection of photons. Also the effects of lateral diffusion of the carriers and the leakage current are neglected. The dielectric materials outside the QW are also taken as of the same kind.

On the boundary of the active region and the DBR mirrors,

$$p^+(L)R_2 = p^-(L), \qquad (5)$$

$$p^-(0)R_1 = p^+(0). \qquad (6)$$

3. Steady-state property

Under steady state, the differentials about t in rate equations are set to zero for all parameters that do not vary with time. Also, in VCSEL, the size of the gain region is quite limited, so R_1 and R_2 are required to be at about 0.99 to lower the cavity loss and get a high quality factor Q. Thus, p^+ and p^- have almost the same value, and we can consider that: $p^+ + p^- \approx 2\sqrt{p^+p^-}$. For $\beta < 0.001$, the effect of spontaneous emission is also negligible. Applying boundary condition, we get

$$\sum_i (\Gamma_{\rm t} g_i - \alpha_{\rm w} + \alpha_{\rm b}) d_i r_i - \alpha_{\rm b} L = \ln \frac{1}{\sqrt{R_1 R_2}}. \qquad (7)$$

Let

$$r_i = \frac{1}{d_i} \int_{{\rm act}_i} 1 + \cos(\theta(z))\,{\rm d}z,$$

where we refer r_i as the longitudinal coupling factor, which is the average of the optical field distributed in the ith QW. r_i, ranging from 0 to 2, reflects how much degree the interference optical field gets coupled with the carriers along the propagation direction. Here $1 + \cos(\theta(z))$ is considered as the distribution function of the optical field in cavity.

After some work, we get the following expression for threshold:

$$J_{\rm th} = J_{\rm N} \exp\left\{ \frac{1}{g_0 \Gamma_{\rm t}} \left[\alpha_{\rm w} - \alpha_{\rm b} + \frac{L}{\sum_i r_i d_i} (\alpha_{\rm m} + \alpha_{\rm b}) \right] \right\}, \qquad (8)$$

where $J_{\rm N} = (n_{\rm N}/\tau_{\rm n}) e \sum_i d_i$ is the basic surface density of current to sustain the lifetime of carriers. $\alpha_{\rm m} = (1/L)\ln(1/\sqrt{R_1R_2})$ is the average of absorption coefficient corresponding to the cavity mirror loss. In Eq. (8), $(L/\sum_i r_i d_i)(\alpha_{\rm m} + \alpha_{\rm b})$ is generally larger than the first term $\alpha_{\rm w} - \alpha_{\rm b}$. Since r_i is in the denominator, we can deduce that when each r_i gets as large as possible, Jth will be lowered.

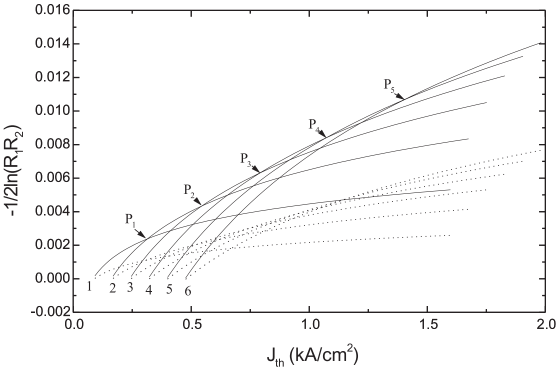

Fig. 1. Cavity facet loss versus threshold current with QW number as a variable parameter. (The solid lines include the effect of interference while the dashed lines do not).

Starting from Eq. (8), we get the curves of threshold current versus the loss of cavity mirror with number of QWs as a variable parameter. Results are shown in Fig. 1 as solid lines. For convenience, we also assume that all QWs are of the same width, which is true for usual lasers. For parameters, refer to Table 1. The point P_n noted in Fig. 1 means that when $\ln(1/\sqrt{R_1R_2})$ lies between P_{n-1} and P_n, VCSEL of n QWs gets the lowest threshold current, i.e., n is the optimal number of QW. The dashed lines in Fig. 1 are obtained with the conventional rate equations not taking interference into account (i.e. let r_i equal to 1). We can see from the figure that the optimal number of

Table 1
Parameters used in the calculations

Coefficient	Symbol	Value
Confinement factor	$\Gamma_{\rm t}$	0.8
Group refractive index	$n_{\rm g}$	4.5
Absorption constant in QW	$\alpha_{\rm w}$	30 $\rm cm^{-1}$
Absorption constant outside QW	$\alpha_{\rm b}$	10 $\rm cm^{-1}$
Gain coefficient	g_0	1.6×10^3 $\rm cm^{-1}$
Transparent carrier density	$n_{\rm th}$	1.37×10^{18} $\rm cm^{-3}$
Wavelength	λ	0.86 μm
Reflectivity of DBR$_1$	R_1	1.00
Reflectivity of DBR$_2$	R_2	0.99
Spontaneous emission factor	β	0.001
Width of QW	d_i	7 nm
Width of barrier	$d_{\rm b}$	7 nm
Lifetime of carrier	$\tau_{\rm n}$	2×10^{-9}
Cavity length	L	0.38 μm
Refractive index in QW	$n_{\rm w}$	3.59
Refractive index outside QW	$n_{\rm b}$	3.385

QWs calculated from the conventional rate equation is larger than that from the modified one. The difference between the two sets of equations is notable, so the introducing of longitudinal coupling factor is quite necessary.

4. Dynamic property

We add the two rate equations of travelling waves for photons and integrate it over the whole cavity, which has integer periods of standing waves. The above assumption of $p^+(z) + p^-(z) \cong 2\sqrt{p^+(z)p^-(z)}$ still holds. In addition, we assume that $p(z)$ varies so slowly with z that it can be taken as invariant with z. Thus the equation gets simplified as

$$\frac{\mathrm{d}p}{\mathrm{d}t} = v_\mathrm{g} p \sum_i \Gamma_\mathrm{t} g_i \frac{r_i d_i}{L} + \Gamma_\mathrm{t} \beta \sum_i \frac{r_i d_i n_i}{2L\tau_\mathrm{n}} - \frac{p}{\tau_\mathrm{p}}, \tag{9}$$

where

$$\tau_\mathrm{p}^{-1} = v_\mathrm{g}\left(\alpha_\mathrm{b} + \frac{2}{L}\frac{1 - R_1 R_2}{(1+R_1)(1+R_2)} + (\alpha_\mathrm{w} - \alpha_\mathrm{b})\sum_i \frac{r_i d_i}{L}\right),$$

where τ_p is the life time of photon, which is now determined by the reflectivity of cavity mirrors, the absorption of dielectric materials and the QWs position in cavity.

The rate equation for carriers remains unchanged

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} = \frac{J(t)}{e\sum_i d_i} - \frac{n_i}{\tau_\mathrm{n}} - 2 v_\mathrm{g} g_i r_i p. \tag{10}$$

In Fig. 2, we show the calculated modulation property for VCSEL with QWs of different number and different ways of distribution in cavity. Results indicate that the modulation bandwidth of periodic gain structure is broader than that of non-periodic gain structure. In periodic gain structure, the photons get a better couple with carriers, on the other hand, the density of photons increase while that of carriers decreases, both permit a broader modulation bandwidth.

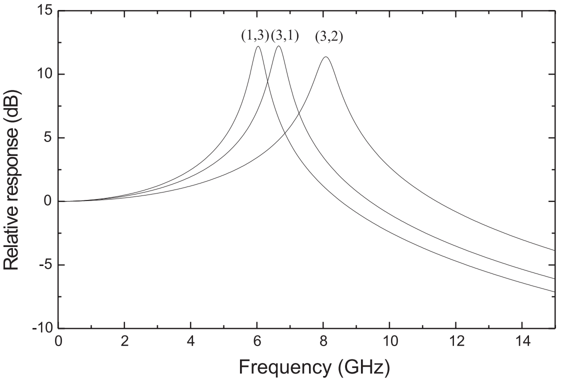

Fig. 2. The modulation property of structures with different number and different distribution of QWs. The first integer in each parentheses represents the number of periods in the whole cavity, while the second is that for the number of QWs in a period. All curves are for structures of the same cavity length.

For either periodic or non-periodic gain structure, when the number of QWs gets larger and the density of carriers decreases, the modulation bandwidth will then be broader.

5. Conclusion

We get a modified travelling wave equation including a term of longitudinal coupling factor, which is of great impact on VCSEL and could not be neglected in the rate equation. Further-more, we discuss the relationship between the threshold current and the optimal number of QWs by employing the above mentioned rate equation. Also this relation is strongly dependent of the cavity loss. We also get an expression for the threshold current of VCSEL, and deduce from it that the larger the longitudinal coupling factor, the lower the threshold current. We analyze the small-signal modulation, and figure out that the periodic gain structure has a relatively broader modulation bandwidth compared to non-periodic gain structure.

Acknowledgements

The project was supported by the Nature Science Fund of China (Grant No. 69896260, 69777005).

References

[1] R.A. Morgan, J.A. Lehman, M.K. Hibbs-Brenner, Y. Liu, J.P. Bristow, SPIE Int. Soc. Opt. Eng. (USA) 3004 (1997) 91.
[2] R.A. Morgan, SPIE. Int. Soc. Opt. Eng. (USA) 3003 (1997) 14.
[3] D.L. Hiffaker, J. Shin, H. Deng, C.C. Lin, D.G. Deppe, B.G. Streetman, Appl. Phys. Lett. 65 (1994) 2642.
[4] R.P. Schneider Jr, J.A. Lott, Appl. Phys. Lett. 63 (1993) 917.
[5] Igor Burgaftman, Jerry R. Meyer, L. Ramdas Ram-Mohan, IEEE J. Quant. Elec. 34 (1) (1998) 147.
[6] Koeth, R. Dietrich, A. Forchel, Appl. Phys. Lett. 72 (1998) 1638.
[7] M.Y.A. Raja, S.R.J. Brueck, M. Osinski, Appl. Phys. Lett. 53 (1988) 1678.
[8] B.-S. Yoo, H.-H. Park, E.-H. Lee, Electronics Letters 30 (13) (1994) 1060.
[9] Govind P. Agraval, Niloy K. Dutta, Semiconductor Lasers, second edition, Van Nostrand Reinhold, USA, 1993.